CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
EET416	NONLINEAR SYSTEMS	PEC	2	1	0	3

Preamble: Most of the systems that we come across are nonlinear. Nonlinear systems exhibit interesting oscillatory behaviours and indeed unexpected phenomena like limit cycles, bifurcation, chaos etc. The course aims in understanding the basic phenomena of limit cycles, determine their existence and non-existence in systems using various theorems. This course also aims to investigate the behaviour of nonlinear systems, analyze their stability using the Lyapunov direct/indirect methods, frequency-domain methods and design various control schemes. For understanding the concepts, a basic mathematical foundation is also built throughout the course.The course will provide the basis for designing controllers for various applications such as aerospace, power systems, robotics, electric drives etc.

Prerequisites: EET 302 Linear Control Systems and EET 401 Advanced Control Systems

Course Outcomes: After the completion of the course the student will be able to:

CO 1	Analyse the qualitative behaviour of nonlinear systems about their equilibrium points.
CO 2	Identify the existence and uniqueness of solutions of nonlinear differential equations, the existence of periodic orbits/limit cycles for nonlinear systems.
CO 3	Analyse the stability of nonlinear systems.
CO 4	Design feedback control systems for nonlinear systems.

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	2	-	-	-	-	-	-	-	-	-	1
CO 2	3	3	-	-	-	-	-	-	-	-	-	1
CO 3	3	3	-	-	-	-	-	-	-	-	-	1
CO 4	3	2	-	-	-	-	-	-	-	-	-	1

Assessment Pattern:

Total Marks	CIE marks	ESE marks	ESE Duration
150	50	100	03 Hrs

Bloom's Category	Continuous Assessment Tests		End Semester Examination
	1	2	
Remember (K1)	10	10	20
Understand (K2)	15	15	30
Apply (K3)	25	25	50
Analyse (K4)			
Evaluate (K5)			
Create (K6)			

End Semester Examination Pattern: There will be two parts; Part A and Part B. **Part A** contains 10 questions with 2 questions from each module having 3 marks for each question. Students should answer all questions.

Part B contains 2 questions from each module of which student should answer anyone. Each question carries 14 marks and can have sub-divisions.

Course Level Assessment Questions:

Course Outcome 1 (CO1)

1. Discuss the characteristics of non-linear systems? (K1, PO1)
2. Model a given nonlinear system. (K2, PO1, PO12)
3. Identify and classify the equilibrium solutions of nonlinear systems. (K2, PO1)
4. Analyse the qualitative behaviour of a given system about its equilibrium points and plot a rough sketch of the phase portrait. (K3, PO2, PO12)
5. What are bifurcations? (K1, PO1)
6. Problems to identify the type of bifurcation. (Saddle-node and Pitchfork only) (K2, PO1)

Course Outcome 2 (CO2):

1. Identify the existence of limit cycles using the Poincare Bendixson theorem. (K3, PO2, PO12)
2. Identify the non-existence of limit cycles using Bendixson's theorem. (K3, PO2, PO12)
3. Problems to check the existence and uniqueness of initial value problems. (K2, PO2)

Course Outcome 3 (CO3):

1. Explain the concept of stability (local and global), instability in the sense of Lyapunov. (K2, PO1)
2. Apply Lyapunov direct/indirect methods to analyze the stability of nonlinear systems. (K3, PO2, PO12)
3. Analyze the stability using LaSalle's invariance theorem. (K3, PO2, PO12)
4. Construct Lyapunov functions using Variable gradient and Krasovskii's method. (K3, PO2)
5. Explain memoryless systems and passivity. (K1, PO1)
6. Examine whether a given system transfer function is positive real or not. (K2, PO1)
7. Explain sector nonlinearity and absolute stability. (K1, PO1)
8. Define KYP Lemma (without proof). (K1, PO1)
9. Examine the stability of the sector nonlinearity using Circle criterion. (K3, PO2)
10. Explain Popov criterion for stability. (K1, PO1)

Course Outcome 4 (CO5):

1. Define feedback control problem - state feedback and output feedback. (K1, PO1)
2. Use state feedback control law for stabilizing a given system. (K2, PO1)
3. Explain the concept of input-state and input-output linearization. (K1, PO1)
4. Examine whether a given system is input-output linearizable. (K3, PO2, PO12)
5. Explain stabilization via integral control. (K1, PO1)

Model Question Paper **QP CODE:**			PAGES: 2
Reg.No:_____________ Name:_______________			

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
EIGHTH SEMESTER B.TECH DEGREE EXAMINATION
MONTH & YEAR

Course Code**: EET416**

Course Name: **NONLINEAR SYSTEMS**

Max. Marks: 100 **Duration: 3 Hours**

PART A
Answer all questions
Each question carries 3 marks

1		Qualitatively analyse the following nonlinear system about the equilibrium point $\dot{y}+0.5\,\dot{y}+2\,y+y^2=0$	3
2		What are limit cycles? Give significance and classify them based on stability.	3
3		Define Poincare Index theorem. Check whether there exist periodic orbits for the system defined below using Poincare index theorem. $\dot{y}-y+y^3=0$	3
4		State the conditions for uniqueness and existence of solutions.	3
5		Check the stability of the nonlinear system using Lyapunov direct method. $(x\_1)^{\cdot} = x\_2$ $(x\_2)^{\cdot} = \quad 〖-x\_1 - 3x\_2〗\_$	3
6		What is meant by domain of attraction of a given system?	3
7		What are positive real transfer functions? Check whether G(s)= [s + 2] / [s + 3]	3

		is a positive real transfer function.	
8		Define absolute stability.	3
9		Find the relative degree for the controlled Van der Pol equation with output $y = x_1$ $\dot{x}_1 = x_2$ $\dot{x}_2 = -x_1 + \varepsilon (1 - x_1^2) x_2 + u, \ \epsilon > 0$	3
10		What is the concept of gain scheduling?	3
PART B **(Answer any one full question from each module)**			
Module 1			
11	a)	Find the equilibrium points of the system defined by the system given below and determine the type of each isolated equilibrium point.Also, plot a rough sketch of the qualitative behaviour near the equilibrium points. $\dot{x}_1 = 5x_1 - x_1 x_2$ $\dot{x}_2 = 3x_2 + x_1 x_2 - 3 x_2^2$	7
	b	The nonlinear dynamic equation for a pendulum is given by $ml\ddot{\theta} = -mg\sin(\theta) - kl\dot{\theta}$ where '*l=1*' is the length of the pendulum, '*m*' is the mass of the bob, and θ is the angle subtended by the rod and the vertical axis through the pivot point. '*g*' is the gravitational constant. Choose '*k/m=1*'. Find all the equilibria of the system and determine if the equilibria are stable or not.	7
12	a	What is saddle-node and Pitch fork bifurcation?	6
	b	Obtain the linearized representation of the following system around the origin and check the stability of the linearised system about the origin. $\dot{x}_1 = x_2^2 + x_1 \cos x_2$ $\dot{x}_2 = x_2 + (x_1 + 1)x_1 + x_1 \sin x_2$	8

		Module 2	
13	a	Define a) Bendixson theorem b) Poincare - Bendixson theorem	6
	c	Check whether the following functions are locally Lipchitz. Give reasons for your claim. (i) $f(x,y) = 2xy^{1/3}$ for (x,y) = [0,0] (ii) $f(t,x) = 2tx^2$ for (x,y) = [0,3]	8
14	a)	Obtain the Lipschitz constant for (i) $f(t,y) = -3y + 2$ (ii) $f(t,y) = 2ty\text{^}2$	7
	b	Check whether the system given below has a stable or unstable limit cycle. $((x\_1)^{\cdot} = x\_2 - x\_1 ([x\_1]\text{^}2 + [x\_2]\text{^}2 - 1))^{\cdot}$ $(x\_2)^{\cdot} = -x\_1 - x\_2 ([x\_1]\text{^}2 + [x\_2]\text{^}2 - 1)$	7
		Module 3	
15		Explain the concept of the domain of attraction using an example.	5
	c)	Use variable gradient method to find a suitable Lyapunov function for the system given below $(x\_1)^{\cdot} = -2x\_1$ $(x\_1)^{\cdot} = -2x\_2 + 2x\_1 [x\_2]\text{^}2$	9
16	a	Define stability in the sense of Lyapunov. What is the difference between asymptotic and exponential stability?	6
	b	State LaSalle's invariance principle. Show that the origin is locally asymptotically stable for the following system using LaSalle's principle. $(x\_1)^{\cdot} = x\_2$ $(x\_2 = -3x\_2)^{\cdot} - [x\_1]\text{^}3$	8

Module 4			
17	a)	What is KYP Lemma?	4
	b	State circle criterion. Determine a stability sector from the Nyquist plot of the system using circle criterion. $$G(s) = 4/((s-1)(s/3+1)(s/5+1))$$	10
18	a)	Using circle criterion, find a sector [a,b] for which the following system is absolutely stable. $$G(s) = 1/((s+1)(s+2)(s+3))$$	8
	b	Describe Popov stability criterion.	6
Module 5			
19	a)	Define the following terms (i) Diffeomorphism (ii) Lie derivative	6
	b	Check whether the given system can be input-output linearized for output $y = x\_1$ $$(x\_1)^{\cdot} = x\_1$$ $$(x\_2)^{\cdot} = x\_2 + u$$	8
20	a)	What is input-output linearization?	6
	b	With a suitable feedback control law, linearize the following system $$(x\_1)^{\cdot} = a \sin x\_2$$ $$(x\_2)^{\cdot} = - 〖x\_1〗^2 + u$$	8

Syllabus

Module 1

Introduction and background (7 hours)

Non-linear system characteristics and mathematical modelling of a non-linear system, Classification of equilibrium points, Stability of a nonlinear system based on equilibrium points, Bifurcation (construction not included), Phase plane analysis of nonlinear systems.

Module 2

Nonlinear characteristics (8 hours)

Periodic solution of nonlinear systems and existence of limit cycle, Open sets, closed sets, connected sets, Invariant set theorem, Bendixson's theorem and Poincare-Bendixson criteria, Existence and uniqueness of solutions to nonlinear differential equations (Proofs not required), Lipschitz condition.

Module 3

Stability Analysis (7 hours)

Lyapunov stability theorems (Proofs not required)- local stability - local linearization and stability in the small- region of attraction, the direct method of Lyapunov, Construction of Lyapunov functions - Variable gradient and Krasovskii's methods, La Salles's invariance principle.

Module 4

Analysis of feedback systems (8 hours)

Passivity and loop transformations, KYP Lemma (Proof not required), Absolute stability, Circle Criterion, Popov Criterion.

Module 5

Nonlinear control systems design (8 hours)

Feedback linearization, Input state linearization method, Input-output linearization method, Stabilization - regulation via integral control- gain scheduling.

Text Book:

1. Khalil H. K., "Nonlinear Systems", 3/e, Pearson, 2002
2. Gibson J. E., "Nonlinear Automatic Control", Mc Graw Hill, 1963
3. Slotine J. E. and Weiping Li, "Applied Nonlinear Control", Prentice-Hall, 1991

References:
1. Alberto Isidori, “Nonlinear Control Systems: An Introduction”, Springer-Verlag, 1985.
2. M. Vidyasagar, “Nonlinear Systems Analysis”, Prentice-Hall, India, 1991.
3. Shankar Sastry, “Nonlinear System Analysis, Stability and Control”, Springer, 1999.

Course Contents and Lecture Schedule

No	Topic	No. of Lectures
1	**Introduction and background (7 hours)**	
1.1	Non-linear system characteristics and mathematical modelling of a non-linear system.	2
1.2	Classification of equilibrium points, Stability of a nonlinear system based on equilibrium points.	2
1.3	Bifurcation (construction not included), Phase plane analysis of nonlinear systems.	3
2	**Nonlinear characteristics (8 hours)**	
2.1	Periodic solution of nonlinear systems and existence of limit cycles	2
2.2	Open sets, closed sets, connected sets, Invariant set theorem, Bendixson's theorem and Poincare-Bendixson criteria	4
2.3	Existence and uniqueness of solutions to nonlinear differential equations (Proofs not required), Lipschitz condition.	2
3	**Stability Analysis (7 hours)**	
3.1	Lyapunov stability theorems (Proofs not required)- local stability - local linearization and stability in the small- region of attraction	2
3.2	The direct method of Lyapunov	2
3.3	Construction of Lyapunov functions, La Salles's invariance principle.	3
4	**Analysis of feedback systems (8 hours)**	

4.1	Passivity and loop transformations	2
4.2	KYP Lemma (Proof not required), Absolute stability	2
4.3	Circle Criterion	2
4.4	Popov Criterion	2
5	**Nonlinear control systems design (8 hours)**	
5.1	Feedback linearization	2
5.2	Input state linearization method	2
5.3	Input-output linearization method	2
5.4	Stabilization - regulation via integral control- gain scheduling	2
