

Reg. No. _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION, MAY 2017

MA202: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100

Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

1. a. Given that $f(x) = \frac{k}{2^x}$ is a probability distribution of a random variable that can take on the values $x = 0, 1, 2, 3$ and 4 , find k . Find the cumulative distribution function. (7)
 b. If 6 of the 18 new buildings in a city violate the building code, what is the probability that a building inspector who randomly select 4 of the new buildings will catch
 - i) none of the new buildings that violate the building code
 - ii) one of the new buildings that violate the building code
 - iii) at least two of the new buildings violate the building code (8)
2. a. Prove that binomial distribution with parameters n and p can be approximated to Poisson distribution when n is large and p is small with $np = \lambda$ a constant. (7)
 b. Find the value of k for the probability density $f(x)$ given below and hence find its mean and variance where

$$f(x) = \begin{cases} kx^3 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$
3. a. A random variable has normal distribution with $\mu = 62.4$. Find its standard deviation if the probability is 0.2 that it will take on a value greater than 79.2 (7)
 b. The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with the parameter 50 days. Find the probability that such a camera will
 - i) have to be reset in less than 20 days
 - ii) not have to be reset in at least 60 days. (8)

PART B (MODULES III AND IV)

Answer two full questions.

4. a. Use Fourier integral to show that $\int_0^\infty \frac{\cos x\omega + \omega \sin x\omega}{1 + \omega^2} d\omega = \begin{cases} 0 & \text{if } x < 0 \\ \pi/2 & \text{if } x = 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases} \quad (7)$
 b. Represent $f(x) = \begin{cases} x^2 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$ as a Fourier cosine integral. (8)

5. a. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{otherwise} \end{cases}$ (7)
- b. Find the Laplace transforms of the following
 i) $\cos t - t \sin t$ ii) $4t e^{-2t}$ (8)
6. a. Find the inverse Laplace transform of the following
 i) $\frac{2s+1}{s^2+2s+5}$ ii) $\frac{(2s-10)}{s^3} e^{-5s}$ (8)
- b. Solve $y'' + 2y' + 5y = 25t$, $y(0) = -2$, $y'(0) = -2$ using Laplace transforms (7)

PART C (MODULES V AND VI)

Answer two full questions.

7. a. Solve $f(x) = x - 0.5 \cos x = 0$ near $x = 0$ by fixed point iteration method. (7)
- b. Solve $f(x) = 2x - \cos x = 0$ by Newton Raphson's method (7)
- c. Find $f(9.2)$ from the values given below by Lagrange's interpolation formula
- | x | 8 | 9 | 9.5 | 11 |
|--------|----------|----------|----------|----------|
| $f(x)$ | 2.197225 | 2.251292 | 2.397895 | 2.079442 |
- (6)
8. a. Given $(x_j, f(x_j)) = (0.2, 0.9980), (0.4, 0.9686), (0.6, 0.8443), (0.8, 0.5358), (1, 0)$, find $f(0.7)$ based on 0.2, 0.4, and 0.6 using Newton's interpolation formula. (10)
- b. Solve $10x_1 + x_2 + x_3 = 6$, $x_1 + 10x_2 + x_3 = 6$, $x_1 + x_2 + 10x_3 = 6$ by Gauss-Seidel iteration method starting at $x_1 = 1$, $x_2 = 1$ and $x_3 = 1$ correct to 4 digits. (10)
9. a. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ with 4 subintervals by Simpson's rule and compare it with the exact solution. (7)
- b. Solve $y' = y$, $y(0) = 1$ by Euler method to find $y(1)$ with $h = 0.2$ (7)
- c. Solve $y' = 1 + y^2$, $y(0) = 0$ by fourth order Runge-Kutta method with $h = 0.1$, 5 steps. (6)
