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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks

Marks

- 1 a) Let $f(z) = u(x, y) + iv(x, y)$ be defined and continuous in some neighbourhood of a point $z = x + iy$ and differentiable at z itself. Then prove that the first order partial derivatives of u and v exist and satisfy the Cauchy – Riemann equations. (7)
- b) Prove that $u = \sin x \cosh y$ is harmonic. Hence find its harmonic conjugate. (8)
- 2 a) Find the image of the region $\left|z - \frac{1}{3}\right| \leq \frac{1}{3}$ under the transformation $w = \frac{1}{z}$. (8)
- b) Find a linear fractional transformation which maps $-1, 0, 1$ onto $1, 1 + i, 1 + 2i$. (7)
- 3 a) Check whether the function $f(z) = \begin{cases} \frac{\operatorname{Re}(z^2)}{|z|^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ is continuous at $z = 0$. (7)
- b) Find the image of the x-axis under the linear fractional transformation $w = \frac{z+1}{2z+4}$. (8)

PART B

Answer any two full questions, each carries 15 marks

- 4 a) Evaluate $\int_C \operatorname{Im}(z^2) dz$ where C is the triangle with vertices $0, 1, i$ counter-clockwise. (7)
- b) Using Cauchy's Integral Formula, evaluate $\int_C \frac{z^2}{z^3 - z^2 - z + 1} dz$ where c is taken counter-clockwise around the circle: (8)
 - i) $|z + 1| = \frac{3}{2}$
 - ii) $|z - 1 - i| = \frac{\pi}{2}$
- 5 a) Determine and classify the singular points for the following functions: (7)
 - i) $f(z) = \frac{\sin z}{(z - \pi)^2}$
 - ii) $g(z) = (z + i)^2 e^{\left(\frac{1}{z+i}\right)}$
- b) Evaluate $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3} dx$. (8)
- 6 a) Evaluate $\int_C \frac{\tan z}{z^2 - 1} dz$ counter clockwise around $c: |z| = \frac{3}{2}$ using Cauchy's Residue Theorem. (7)
- b) Find all Taylor series and Laurent series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with centre 0 in (8)
 - i) $|z| < 1$
 - ii) $1 < |z| < 2$.

PART C

Answer any two full questions, each carries 20 marks

- 7 a) Solve the system of equations by Gauss Elimination Method: (8)
 $3x + 3y + 2z = 1, \quad x + 2y = 4, \quad 10y + 3z = -2, \quad 2x - 3y - z = 5.$
- b) Prove that the vectors $(1, 1, 2), (1, 2, 5), (5, 3, 4)$ are linearly dependent. (6)
- c) Prove that the set of vectors $V = \{(v_1, v_2, v_3) \in \mathbb{R}^3 : -v_1 + v_2 + 4v_3 = 0\}$ a vector space over the field \mathbb{R} . Also find the dimension and the basis. (6)
- 8 a) Find the Eigen values and the corresponding Eigen vectors of (8)
 $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$
- b) What kind of conic section is given by the quadratic form $7x_1^2 + 6x_1x_2 + 7x_2^2 = 200$. Also find its equation. (6)
- c) Determine whether the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ symmetric, skew-symmetric or orthogonal. (6)
- 9 a) Reduce the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ to Row Echelon Form and hence find its rank. (8)
- b) Diagonalize $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ (12)
