

Reg. No. _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, MARCH 2017

MA 201: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer any 2 questions

1. a. Check whether the following functions are analytic or not. Justify your answer.
 - i) $f(z) = z + \bar{z}$ (4)
 - ii) $f(z) = |z|^2$ (4)
 b. Show that $f(z) = \sin z$ is analytic for all z . Find $f'(z)$ (7)
2. a. Show that $v = 3x^2y - y^3$ is harmonic and find the corresponding analytic function

$$f(z) = u(x, y) + iv(x, y)$$
 (8)
 b. Find the image of $0 < x < 1, \frac{1}{2} < y < 1$ under the mapping $w = e^z$ (7)
3. a. Find the linear fractional transformation that carries $z_1 = -2, z_2 = 0$ and $z_3 = 2$ on to the points $w_1 = \infty, w_2 = 1/4$ and $w_3 = 3/8$. Hence find the image of x-axis. (7)
 b. Find the image of the rectangular region $-\pi \leq x \leq \pi, a \leq y < b$ under the mapping $w = \sin z$ (8)

PART B

Answer any 2 questions

4. a. Evaluate $\int_C |z| dz$ where
 - i) C is the line segment joining $-i$ and i (3)
 - ii) C is the unit circle in the left of half plane (4)
 b. Verify Cauchy's integral theorem for z^2 taken over the boundary of the rectangle with vertices $-1, 1, 1+i, -1+i$ in the counter clockwise sense. (8)
5. a. Find the Laurent's series expansion of $f(z) = \frac{1}{1-z^2}$ which is convergent in
 - i) $|z - 1| < 2$ (4)
 - ii) $|z - 1| > 2$ (4)
 b. Determine the nature and type of singularities of
 - i) $\frac{e^{-z^2}}{z^2}$ (3)

$$\text{ii) } z \sin\left(\frac{1}{z}\right) \quad (4)$$

$$6. \text{ a. Use residue theorem to evaluate } \int_C \frac{30z^2 - 23z + 5}{(2z-1)^2(3z-1)} dz \text{ where } C \text{ is } |z|=1 \quad (7)$$

$$\text{b. Evaluate } \int_0^\infty \frac{1}{(1+x^2)^2} dx \text{ using residue theorem.} \quad (8)$$

PART C

Answer any 2 questions

7. a. Solve the following by Gauss elimination

$$y + z - 2w = 0, \quad 2x - 3y - 3z + 6w = 2, \quad 4x + y + z - 2w = 4 \quad (6)$$

b. Reduce to Echelon form and hence find the rank of the matrix

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix} \quad (6)$$

$$\text{c. Find a basis for the null space of } \begin{bmatrix} 2 & -2 & 0 \\ 0 & 4 & 8 \\ 2 & 0 & 4 \end{bmatrix} \quad (8)$$

8. a. i) Are the vectors $(3 \ -1 \ 4)$, $(6 \ 7 \ 5)$ and $(9 \ 6 \ 9)$ linearly dependent or independent? Justify your answer. (5)

ii) Is all vectors (x, y, z) in \mathbb{R}^3 with $y - x + 4z = 0$ form a vector space over the field of real numbers? Give reasons for your answer. (5)

b. i) Find a matrix C such that $Q = \mathbf{x}^T C \mathbf{x}$ where

$$Q = -3x_1^2 + 4x_1x_2 - x_2^2 + 2x_1x_3 - 5x_3^2 \quad (4)$$

ii) Obtain the matrix of transformation

$$y_1 = \cos \theta x_1 - \sin \theta x_2, \quad y_2 = \sin \theta x_1 + \cos \theta x_2$$

Prove that it is orthogonal. Obtain the inverse transformation. (6)

9. a. Find the eigenvalues, eigenvectors and bases and dimensions for each Eigen space of

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \quad (10)$$

b. Find out what type of conic section, the quadratic form $17x_1^2 - 30x_1x_2 + 17x_2^2 = 128$ and transform it to principal axes. (10)