

Reg. No:.....

Name:.....

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY****FIRST SEMESTER B.TECH DEGREE EXAMINATION, FEBRUARY 2017****MA101: CALCULUS**

Max. Marks: 100

Duration: 3 Hours

**PART A***(Answer All Questions and each carries 5 marks)*

1. a) Test the convergence of  $\sum_{k=1}^{\infty} \frac{99^k}{k!}$  (2)  
b) Test the convergence of  $\sum_{k=1}^{\infty} \frac{1}{3^{k+1}}$ . (3)
2. a) Find the slope of the sphere  $x^2+y^2+z^2=1$  in the y- direction at  $(\frac{2}{3}, \frac{1}{3}, \frac{-2}{3})$ . (2)  
b) Find the critical points of the function  $f(x,y) = 2xy - x^3 - y^3$ . (3)
3. a) Find the velocity at time  $t = \pi$  of a particle moving along the curve  
 $\vec{r}(t) = e^t \sin t \, i + e^t \cos t \, j + t \, k$ . (2)  
b) Find the directional derivative of  $f(x,y) = xe^y - ye^x$  at the point  $P(0,0)$  in the direction of  $5i - 2j$ . (3)
4. a) Change the order of integration in  $\int_0^1 \int_{y^2}^{\sqrt{2-y^2}} f(x,y) dx dy$ . (3)  
b) Find the area of the region enclosed by  $y = x^2$  and  $y = x$ . (2)
5. a) Find the divergence of the vector field  $f(x,y,z) = x^2y \, i + 2y^3z \, j + 3z \, k$ . (2)  
b) Find the work done by  $\vec{F} = xy \, i + x^3j$  on a particle that moves along the curve  $y^2=x$  from  $(0,0)$  to  $(0,1)$ . (3)
6. a) Using Green's theorem to evaluate  $\int_C 2xy \, dx + (x^2 + x) \, dy$  where C is the triangle with vertices  $(0,0)$ ,  $(1,0)$  and  $(1,1)$ . (2)  
b) Use Stoke's theorem to evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $\vec{F} = (x-2y)i + (y-z)j + (z-x)k$  and C is the circle  $x^2+y^2 = a^2$  in the xy plane with counter clockwise orientation looking down the positive z- axis. (3)

## PART B

## MODULE I (Answer Any Two Questions)

7. a) Test the convergence of the following series (5)

i)  $\sum_{k=1}^{\infty} \frac{(k+4)!}{4!k! 4^k}$       ii)  $\sum_{k=2}^{\infty} \left(\frac{4k-5}{2k+1}\right)^k$

8. Use the alternating series test to show that the series  $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(k+3)}{k(k+1)}$  converge. (5)

9. Find the Taylor's series of  $f(x) = x \sin x$  about the point  $x = \frac{\pi}{2}$ . (5)

## MODULE II (Answer Any Two Questions)

10. Find the local linear approximation  $L$  to  $f(x,y) = \ln(xy)$  at  $P(1,2)$  and compare the error in approximating  $f$  by  $L$  at  $Q(1.01, 2.01)$  with the distance between  $P$  and  $Q$ . (5)

11. Show that the function  $f(x,y) = 2 \tan^{-1}(y/x)$  satisfies the Laplace's equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0. \quad (5)$$

12. Find the relative minima of  $f(x,y) = 3x^2 - 2xy + y^2 - 8y$ . (5)

## MODULE III (Answer Any Two Questions)

13. Find the unit tangent vector and unit normal vector to  $\vec{r} = 4 \cos t \mathbf{i} + 4 \sin t \mathbf{j} + t \mathbf{k}$  at  $t = \frac{\pi}{2}$ . (5)

14. Suppose a particle moves through 3- space so that its position vector at time  $t$  is

$$\vec{r} = t \mathbf{i} + t^2 \mathbf{j} + t^3 \mathbf{k}. \text{ Find the scalar tangential component of acceleration at the time } t=1. \quad (5)$$

15. Given that the directional derivative of  $f(x,y,z)$  at  $(3,-2, 1)$  in the direction of  $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$  is -5 and that  $\|\nabla f(3, -2, 1)\| = 5$ . Find  $\nabla f(3, -2, 1)$ . (5)

## MODULE IV (Answer Any Two Questions)

16. Evaluate the integral  $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dx dy$  by reversing the order of integration. (5)

17. Evaluate  $\int_0^1 \int_0^{y^2} \int_{-1}^z z dx dy$ . (5)

18. Find the volume of the solid in the first octant bounded by the co-ordinate planes and the plane  $x+2y+z = 6$ . (5)

## MODULE V (Answer Any Three Questions)

19. Let  $\vec{r} = xi + yj + zk$  and let  $r = \|\vec{r}\|$  and  $f$  be a differentiable function of one variable

show that  $\nabla f(r) = \frac{f'(r)}{r} \vec{r}$ . (5)

20. Evaluate the line integral  $\int_C [-y dx + x dy]$  along  $y^2 = 3x$  from (3,3) to (0,0). (5)

21. Show that  $\vec{F}(x,y) = (\cos y + y \cos x) i + (\sin x - x \sin y) j$  is a conservative vector field.

Hence find a potential function for it. (5)

22. Show that the integral  $\int_C (3x^2 e^y dx + x^3 e^y dy)$  is independent of the path and hence

evaluate the integral from (0,0) to (3,2). (5)

23. Find the work done by the force field  $\vec{F} = xy i + yz j + xz k$  on a particle that moves

along the curve  $C: \vec{r}(t) = t i + t^2 j + t^3 k$  where  $0 \leq t \leq 1$ . (5)

## MODULE VI (Answer Any Three Questions)

24. Use Green's theorem to evaluate the integral  $\int_C (x \cos y dx - y \sin x dy)$  where

$C$  is the square with vertices (0,0),  $(\pi,0)$ ,  $(\pi,\pi)$  and  $(0,\pi)$ . (5)

25. Evaluate the surface integral  $\int_{\sigma} \int z^2 ds$  where  $\sigma$  is the portion of the

cone  $z = \sqrt{x^2 + y^2}$  between the planes  $z = 1$  and  $z = 3$ . (5)

26. Use divergence theorem to find the outward flux of the vector field  $\vec{F} = 2x i + 3y j + z^2 k$

across the unit cube  $x = 0, x = 1, y = 0, y = 1, z = 0$  and  $z = 1$ . (5)

27. Use Stoke's theorem to evaluate the integral  $\int_C \vec{F} \cdot d\vec{r}$  where  $\vec{F} = (x-y) i + (y-z) j + (z-x) k$

and  $C$  is the boundary of the portion of the plane  $x+y+z = 1$  in the first octant. (5)

28. Use Stoke's theorem to evaluate the integral  $\int_C \vec{F} \cdot d\vec{r}$  where  $\vec{F} = 2z i + 3x j + 5y k$  and

$C$  is the boundary of the paraboloid  $x^2 + y^2 + z = 4$  for which  $z \geq 0$  and  $C$  is positively oriented. (5)

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