Reg No.:_______________ Name:___________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, JULY 2018

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks Marks

1	a)	Express the repeating decimal 1.454545….as a fraction.	(2)
	b)	Expand $f(x) = sin\pi x$ using Taylor series about $x = \frac{1}{2}$.	(3)
2	a)	Find the slope of the surface $z = \sqrt{3x+2y}$ in the x-direction at the point (1,3).	(2)
	b)	Find a linear approximation of $f(x, y, z) = x^2 - xy + 3\sin z$ at the point (2,1,0).	(3)
3	a)	Determine whether $\vec{r}(t) = 3\sin t\hat{i} + 3t\hat{j}$ is continuous at t = 0.	(2)
	b)	Find the gradient of $f(x, y) = x^2 e^y$ at (1,0).	(3)
4	a)	Evaluate $\int_0^1 \int_0^{\log y} \frac{dxdy}{\log y}$.	(2)
	b)	Using double integral find the area bounded by $x = y^2, x + y = 2$ in the positive quadrant.	(3)
5	a)	Find the divergence of vector field $\bar{V} = x^2 y^2 \hat{i} + 2xy\hat{j}$.	(2)
	b)	Show that the vector field $\bar{F} = 2x(y^2 + z^3)\hat{i} + 2x^2 y\hat{j} + 3x^2 z^2 \hat{k}$ is conservative.	(3)
6	a)	Locate sources or sinks of $\bar{F} = xy\hat{i} - 2xy\hat{j} + y^2\hat{k}$.	(2)
	b)	By Green's theorem evaluate $\oint_C (3x + y)dx + (2x + y)dy$, C is the circle $x^2 + y^2 = a^2$.	(3)

PART B

Module I

Answer any two questions, each carries 5 marks

7	Determine whether $\sum_{k=1}^{\infty} \frac{1}{(3k-1)(3k+2)}$ is convergent.	(5)
8	Test the convergence of $1 + \frac{1.3}{3!} + \frac{1.3.5}{5!} + \frac{1.3.5.7}{7!} + ...$	(5)
9	Expand $f(x) = \frac{1}{x}$ in powers of (x-2). Find the interval of convergence and radius of convergence of the power series obtained.	(5)

Module II

Answer any two questions, each carries 5 marks

10 If $W = f(x-y, y-z, z-x)$, Show that $\frac{\partial W}{\partial x}+\frac{\partial W}{\partial y}+\frac{\partial W}{\partial z}=0$. (5)

11 a) If $f(x,y) = y\sin x + e^{2x}\cos y$, find f_{yxx}. (3)

b) State the second partial test for local(relative)extreme values. (2)

12 An aquarium with rectangular sides and bottom (no top) is to hold 32 liters of water. Find its dimension so that least quantity of material is required for its construction. (5)

Module III

Answer any two questions, each carries 5 marks

13 Find the unit tangent $\hat{T}(t)$ and unit normal $\hat{N}(t)$ to the curve $\overline{r}(t) = e^t \cos t\hat{i} + e^t \sin t\hat{j} + e^t\hat{k}$ at t=0. (5)

14 Suppose the position vector of a moving particle is $\overline{r} = t^2\hat{i} + \frac{1}{3}t^3\hat{j}$, find the displacement and distance traveled over the time interval $2 \le t \le 4$. (5)

15 Find the equation of tangent plane to the surface $x^2 + y^2 + z^2 = 25$ at P(3,0,4).Also find the parametric equation for the normal line to the surface at P. (5)

Module IV

Answer any two questions, each carries 5 marks

16 Evaluate $\iint_R e^{x^2}\,dxdy$, where the region R is given by $2y \le x \le 2$ and $0 \le y \le 1$. (5)

17 Evaluate $\iint r\sin\theta\, dr d\theta$ over the cardioid $r = a(1-\cos\theta)$ above the initial line. (5)

18 Evaluate the triple integral $\iiint xy\sin(yz)\,dV$ over the rectangular box $0 \le x \le \frac{\pi}{2}, 0 \le y \le 1, 0 \le z \le \frac{\pi}{6}$. (5)

Module V

Answer any three questions, each carries 5 marks

19 Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$ where $r = \sqrt{x^2+y^2+z^2}$. (5)

20 Evaluate $\int_C ydx + zdy + xdz$ along the twisted cubic $x = t, y = t^2, z = t^3$ from (0,0,0) to (1,1,1). (5)

21 Find the potential function for the vector field $\overline{F} = (\sin z + y\cos x)\hat{i} + (\sin x + z\cos y)\hat{j} + (\sin y + x\cos z)\hat{k}$. (5)

22 Find the work done in moving a particle in the force field $\overline{F} = (x+y)\hat{i} - x^2\hat{j} + (y+z)\hat{k}$ along the curve defined by $x^2 = 4y, z = x, 0 \le x \le 2$. (5)

23 Show that $\int_C (yz-1)dx + (z + xz + z^2)dy + (y + xy + 2yz)dz$ is independent of the path of integration. Find the scalar potential and the value of the integral from (1,2,2) to (2,3,4). (5)

Module VI

Answer any three questions, each carries 5 marks

24 Use Green's theorem to evaluate $\int_C x^2 y dx + x dy$ where C is the boundary of triangular region enclosed by $y = 0, y = 2x, x = 1$. (5)

25 Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using Green's theorem. (5)

26 Evaluate the surface integral $\iint_\sigma xy dS$ where σ is the portion of the plane $x + y + z = 2$ lying in the first octant. (5)

27 Using divergence theorem evaluate $\iint_S \vec{F}.\hat{n} dS$ where $\vec{F} = 2x\hat{i} + 4y\hat{j} - 3z\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. (5)

28 Use Stokes' theorem to calculate the circulation around the bounding circle $x^2 + y^2 = 9$ and the field $\bar{A} = y\, i - x\, j$ where S is the disk of radius 3 centered at origin in the XY-plane. (5)

****