

Reg.No.....

Name.....

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY****FIRST SEMESTER B.TECH DEGREE (SUPPLEMENTARY) EXAMINATION,  
FEBRUARY 2017 (2015 ADMISSION)**Course Code: **MA 101**Course Name: **CALCULUS**

Max.Marks : 100

Duration : 3 Hours

**PART A****(Answer all questions. Each question carries 3 marks)**

- 1) Show that the series  $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$  converges.
- 2) Classify the surface  $z = (x - 1)^2 + (y + 2)^2 + 3$
- 3) Find the Maclaurin series for  $\cos x$
- 4) Evaluate  $\lim_{(x,y) \rightarrow (-1,2)} \frac{xy}{x^2 + y^2}$
- 5) Convert the cylindrical co-ordinate into rectangular co ordinate of  $(4, \pi/3 - 3)$ .
- 6) Find the slope of the surface  $z = xy^2$  in the  $x$  direction at the point  $(2,3)$ .
- 7) Find the directional derivative of  $f = x^2y - yz^3 + z$  at  $(1,-2,0)$  in the direction of  $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$
- 8) Find the unit normal to the surface  $xy + xz + yz = c$  at  $(-1,2,3)$
- 9) Evaluate  $\int_1^a \int_1^b x^2 y \, dx dy$
- 10) Find the area of the region R enclosed by  $y = 1, y = 2, x = 0, x = y$ .

**PART B****(Answer any 2 questions. Each question carries 7 marks)**

- 11) Test the absolute convergence of  $\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{4^n}$
- 12) Determine the Taylor's series expansion of  $f(x) = \sin x$  at  $x = \pi/2$ .
- 13) Test the convergence of  $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$

**(Answer any 2 questions. Each question carries 7 marks)**

- 14) Find the equation of the paraboloid  $z = x^2 + y^2$  in the cylindrical and spherical coordinates.
- 15) Find  $F(f(x), g(y), h(z))$  if  $F(x, y, z) = y e^{xyz}$ ,  $f(x) = x^2$ ,  $g(y) = y + 1$ ,  $h(z) = 2z^2$
- 16) By converting into polar coordinate evaluate  $\lim_{(x,y) \rightarrow (0,0)} \sqrt{x^2 + y^2} \ln \left( (x^2 + y^2)^2 \right)$

**(Answer any 2 questions. Each question carries 7 marks)**

- 17) Find the local linear approximation  $L$  of  $f(x, y, z) = xyz$  at the point  $P(1, 2, 3)$ . Compare the error in approximating  $f$  by  $L$  at the point  $Q(1.001, 2.002, 3.003)$  with the distance  $PQ$ .
- 18) Find the relative extrema of  $f(x, y) = 3x^2 - 2xy + y^2 - 8y$
- 19) If  $f$  is a differentiable function of three variables and suppose that

$$w = f(x - y, y - z, z - x) \quad \text{Show that } \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$$

**(Answer any 2 questions. Each question carries 7 marks)**

- 20) Suppose that a particle moves along a curve in 3-space so that its position vector at time  $t$  is  $r(t) = 4\cos \pi t \mathbf{i} + 4\sin \pi t \mathbf{j} + t \mathbf{k}$ . Find the distance travelled and the displacement of the particle during the time interval  $1 \leq t \leq 5$
- 21) A particle is moving along the curve,  $\vec{r} = (t^3 - 2t)\vec{i} + (t^2 - 4)\vec{j}$  where  $t$  denotes the time. Find the scalar tangential and normal components of acceleration at  $t = 1$ . Also find the vector tangential and normal components of acceleration at  $t = 0$ .
- 22) Find the arc length of the parametric curve  $x = 5\cos t$ ,  $y = 5\sin t$ ,  $z = 2t$ ;  $0 \leq t \leq \pi$

**(Answer any 2 questions. Each question carries 7 marks)**

- 23) Evaluate the integral by converting into polar co ordinates  $\int_0^{2\sqrt{4-x^2}} \int_0^x (x^2 + y^2) dy dx$
- 24) Using triple integral to find the volume bounded by the cylinder  $x^2 + y^2 = 4$  and the planes  $z = 0$  and  $y + z = 3$
- 25) Change the order of integration and evaluate  $\int_0^1 \int_x^1 \frac{x}{x^2 + y^2} dx dy$

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