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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries5 marks. Marks

1 a) Test the convergence of the series $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{2k-1}}$. (2)

b) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{2n+3}$. (3)

2 a) Find the Slope of the surface $z = xe^{-y} + 5y$ in the y-direction at the point (4,0). (2)

b) Find the derivative of $z = \sqrt{1 + x - 2xy^4}$ with respect to t along the path $x = \log t,\ y = 2t.$ (3)

3 a) Find the directional derivative of $f = x^2 y - yz^3 + z$ at $(-1,2,0)$ in the direction of $a = 2i + j + 2k.$ (2)

b) Find the unit tangent vector and unit normal vector to $r(t) = 4\cos t i + 4\sin t j + tk$ at $t = \frac{\pi}{2}.$ (3)

4 a) Evaluate $\int_0^{\log 3} \int_0^{\log 2} e^{x+2y}\, dy dx.$ (2)

b) Evaluate $\iint_R xy\, dA$, where R is the region bounded by the curves $y = x^2$ and $x = y^2.$ (3)

5 (a) Find the divergence and curl of the vector $F(x,y,z) = yz\, i + xy^2 j + yz^2 k.$ (2)

(b) Evaluate $\int_C (3x^2 + y^2)dx + 2xy\, dy$ along the circular arc C given by $x = \cos t, y = \sin t$ for $0 \le t \le \frac{\pi}{2}.$ (3)

6 (a) Use line integral to evaluate the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$ (2)

(b) Evaluate $\int_C (x^2 - 3y)dx + 3xdy$, where C is the circle $x^2 + y^2 = 4.$ (3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

7 Test the convergence or divergence of the series $\sum_{n=1}^{\infty} (\frac{n}{n+1})^{n^2}$. (5)

8 Test the absolute convergence of $\sum_{k=1}^{\infty}(-1)^{k+1}\dfrac{(2k)!}{(3k-2)!}$. (5)

9 Find the Taylor series for $\dfrac{1}{1+x}$ at $x=2$. (5)

Module 1I

Answer any two questions, each carries 5 marks.

10 Find the local linear approximation L to $f(x,y)=\log(xy)$ at P(1,2) and compare the error in approximating f by L at Q(1.01, 2.01) with the distance between P and Q. (5)

11 Let $w=4x^2+4y^2+z^2, x=\rho\sin\phi\cos\theta, y=\rho\sin\phi\sin\theta, z=\rho\cos\phi$. Find $\dfrac{\partial w}{\partial\rho},\dfrac{\partial w}{\partial\phi}$ and $\dfrac{\partial w}{\partial\theta}$. (5)

12 Locate all relative extrema and saddle points of $f(x,y)=4xy-x^4-y^4$. (5)

Module 1II

Answer any two questions, each carries 5 marks.

13 Find the equation of the tangent plane and parametric equation for the normal line to the surface $x^2+y^2+z^2=25$ at the point $(3,0,4)$. (5)

14 A particle is moving along the curve $r(t)=(t^3-2t)i+(t^2-4)j$ where t denotes the time. Find the scalar tangential and normal components of acceleration at $t=1$. Also find the vector tangential and normal components of acceleration at $t=1$. (5)

15 The graphs of $r_1(t)=t^2i+tj+3t^3k$ and $r_2(t)=(t-1)i+\frac{1}{4}t^2j+(5-t)k$ are intersect at the point $P(1,1,3)$ .Find, to the nearest degree, the acute angle between the tangent lines to the graphs of $r_1(t)\,\&\,r_2(t)$ at the point $P(1,1,3)$. (5)

Module 1V

Answer any two questions, each carries5 marks.

16 Change the order of integration and evaluate $\int_0^1\int_{4x}^4 e^{-y^2}\,dydx$. (5)

17 Use triple integral to find the volume bounded by the cylinder $x^2+y^2=9$ and between the planes $z=1$ and $x+z=5$. (5)

18 Find the area of the region enclosed between the parabola $y=\dfrac{x^2}{2}$ and the line $y=2x$. (5)

Module V

Answer any three questions, each carries5 marks.

19 Determine whether $F(x,y)=(\cos y+y\cos x)i+(\sin x-x\sin y)j$ is a conservative vector field. If so find the potential function for it. (5)

20 Show that the integral $\int_{(1,1)}^{(3,3)}(e^x\log y-\dfrac{e^y}{x})dx+(\dfrac{e^x}{y}-e^y\log x)dy$,where x and y are positive is independent of the path and find its value. (5)

21 Find the work done by the force field $F(x,y,z)=xy\,i+yz\,j+xz\,k$ on a particle that moves along the curve $C:r(t)=ti+t^2j+t^3k\,(0\le t\le 1)$. (5)

22 Let $\bar{r} = xi + yj + zk$ and $r = \|\bar{r}\|$, let f be a differentiable function of one variable, then show that $\nabla f(r) = \frac{f'(r)}{r}\bar{r}$. (5)

23 Find $\nabla.(\nabla \times F)$ and $\nabla \times (\nabla \times F)$ where $F(x,y,z) = e^{xz}i + 4xe^{y}j - e^{yz}k$. (5)

Module VI

Answer any three questions, each carries5 marks.

24 Use Green's Theorem to evaluate $\int_C \log(1+y)dx - \frac{xy}{(1+y)}dy$,where C is the triangle with vertices (0,0), (2,0) and (0,4). (5)

25 Evaluate the surface integral $\iint_\sigma xzds$,where σ is the part of the plane $x + y + z = 1$ that lies in the first octant. (5)

26 Using Stoke's Theoremevaluate $\int_C F.dr$ where $F(x,y,z) = xzi + 4x^2y^2j + yxk$, C is the rectangle $0 \le x \le 1, 0 \le y \le 3$ in the plane $z = y$. (5)

27 Using Divergence Theorem evaluate $\iint_\sigma \overline{F}.n\,ds$ where $F(x,y,z) = x^3i + y^3j + z^3k$, σ is the surface of the cylindrical solid bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 4$. (5)

28 Determine whether the vector fields are free of sources and sinks. If it is not, locate them (5)

(i) $(y+z)i - xz^3j + x^2 \sin y\,k$ (ii) $xy\,i - 2xy\,j + y^2k$

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